Disorder Chaos in Diluted Mixed p-spin Model

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Diluted mixed p-spin model

- For each $2 \leq p \leq \Delta$ and $\alpha_p > 0$,

$$E_p = \left\{ \text{Indep. choose each } p\text{-set in } [N] \text{ with probability } \frac{\alpha_p N}{\binom{N}{p}} \right\}.$$

- *Hamiltonian*

$$H_J(\sigma) = \sum_{2 \leq p \leq \Delta} \sum_{e \in E_p} J_e \sigma_e, \quad \sigma \in \{\pm 1\}^N,$$

where $\sigma_e = \prod_{i \in V(e)} \sigma_i$ and $(J_e)_{e \in \cup_{2 \leq p \leq \Delta} E_p}$ are i.i.d. symmetric random variables.

Disorder chaos

- Discrete perturbation $J(t) = (1-B)J + BJ'$ for i.i.d. J and J', and independently, $B \sim \mathrm{Ber}(1-e^{-t})$, $t \geq 0$.
- Sample u.a.r. the *ground states* (σ, τ) for H_J and $H_{J(t)}$.
- *Overlap*: $R(\sigma, \tau) = N^{-1} \sum_{i=1}^{N} \sigma_i \tau_i$.
- *Disorder chaos* if for any $t > 0$,

$$\lim_{N \to \infty} \mathbb{E} R(\sigma, \tau)^2 = 0.$$

- Measures chaotic sensitivity of the system's configurations to small perturbation in its disorder.

Main result

Assume $\lambda := \sum_{2\leq p\leq \Delta} p(p-1)\alpha_p > 1$.

Theorem (Chen, K., Sen ('24))

Consider the diluted mixed p-spin model. Let $t \geq 0$. There exists a constant $C > 0$ depending only on $(\alpha_p)_{2\leq p\leq \Delta}$ such that

$$\mathbb{E}R(\sigma,\tau)^2 \leq \frac{C}{N^{t/(t+2\log\lambda)}}.$$

Remark.

- $\lambda > 1$ ensures that the model is non-trivial.
- Any symmetric disorder is possible.
- Unlike Chen-Panchenko ('18), includes odd p interactions and no restriction on $(\alpha_p)_{2\leq p\leq \Delta}$.
- Based on a suitable extension of the Hermite spectral method in Chatterjee ('23).