Preventing Finite-Time Blowup for Stochastic Reaction-Diffusion Equations with Constrained Forcing

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Equation of interest

Bounded spatial domain $x \in D \subset \mathbb{R}^d$

$$\begin{cases} \frac{\partial u}{\partial t}(t,x) = \mathcal{A}u(t,x) + f(u(t,x)) + \sigma(u(t,x))\dot{W}(t,x), t > 0, x \in D \\ u(t,x) = 0, \; x \in \partial D \\ u(0,x) = u_0(x) \end{cases}$$

- Self-adjoint second-order elliptic differential operator $\mathcal{A}$.
- Nonlinear reaction term $f(u(t,x))$ and multiplicative noise term $\sigma(u(t,x))\dot{W}(t,x)$
  - Both f, σ are infinite near ± 1.
  - $f(u) \approx (1-|u|)^{-\beta}$ and $\sigma(u) \approx (1-|u|)^{-\gamma}$.

Ivanhoe and Salins (2024)

Growth Rates

Assume that there exists $K_1, K_2, c_0, \beta > 0, \gamma > 0$ such that for $1 > |u| > c_0$

$$f(u)\text{sign}(u) \leq -K_1(1-|u|)^{-\beta}$$

$$|\sigma(u)| \leq K_2(1-|u|)^{-\gamma}$$

If $\gamma + 1 < \frac{(1-\eta)(\beta+1)}{2}$, then $\mathbb{P}(\tau < \infty) = 0$

Where we define the blow-up time

$$\tau = \inf\{t \geq 0 : ||u(t,\cdot)||_{L^\infty(D)} = 1\},$$

assuming $||u(0,\cdot)||_{L^\infty(D)} < 1$.

Future Directions

The current literature has addressed polynomially-growing f, σ on the whole space. We have examined the case of $(1 - |u|)^{-\beta}$ growth for f, σ on an interval.

- Is our bound optimal?
- Can these arguments be generalized to arbitrary growing f, σ on the whole space?
- What about arbitrary growth rates for f, σ on an arbitrary constrained potential?