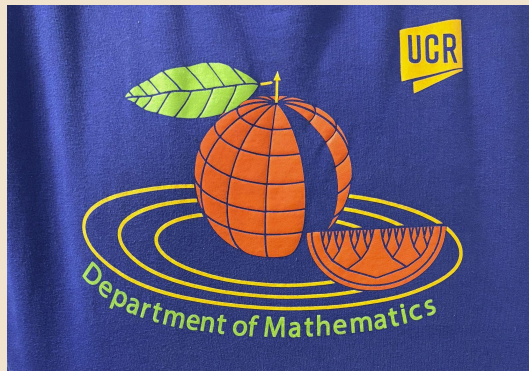


# p-adic Brownian Motion

Rahul Rajkumar (UC Riverside)



# What and Why p-adic?

- The *other* real numbers (Ostrowski's Theorem in Number Theory)
- An idealized model for hierarchical and ultrametric settings
  - Planck-scale physics (The Volovich Hypothesis)
  - Spin glasses
  - Contagion modeling



**ME**  
Can we stop and get some **Brownian motion** ?

**MY ADVISOR:** We have **Brownian motion** at home

**Brownian motion** at home:

**P-ADIC  
ANALYSIS**

# Can p-adic Brownian Motion Teach Us About Brownian Motion?

- p-adic Brownian motions have component processes which are p-adic Brownian motion, like the classical theory
  - (R & Weisbart, 2023)
- p-adic Brownian motion is finitely approximable by scaling limits of random walks, like the classical theory
  - (Weisbart & co. 2016 - 2024; R and Weisbart 2025+)
- p-adic Brownian motions have component processes which are stochastically *dependent*, unlike the classical theory
  - (R & Weisbart 2023)

# Select References

- Bakken, Erik Makino, Trond Digernes, and David Weisbart. “Brownian Motion and Finite Approximations of Quantum Systems over Local Fields.” *Reviews in Mathematical Physics* 29, no. 05 (June 2017): 1750016. <https://doi.org/10.1142/S0129055X17500167>.
- Bakken, Erik, and David Weisbart. “Continuous Time P-Adic Random Walks and Their Path Integrals.” *Journal of Theoretical Probability* 32, no. 2 (June 1, 2019): 781–805. <https://doi.org/10.1007/s10959-018-0831-3>.
- ———. “P-Adic Brownian Motion as a Limit of Discrete Time Random Walks.” *Communications in Mathematical Physics* 369, no. 2 (July 1, 2019): 371–402. <https://doi.org/10.1007/s00220-019-03447-y>.
- Pierce, Tyler, Rahul Rajkumar, Andrea Stine, David Weisbart, and Adam M. Yassine. “Brownian Motion in a Vector Space over a Local Field Is a Scaling Limit.” *Expositiones Mathematicae* 42, no. 6 (December 2024): 125607. <https://doi.org/10.1016/j.exmath.2024.125607>.
- Rajkumar, Rahul, and David Weisbart. “Components and Exit Times of Brownian Motion in Two or More P-Adic Dimensions.” *Journal of Fourier Analysis and Applications* 29, no. 6 (November 20, 2023): 75. <https://doi.org/10.1007/s00041-023-10053-z>.
- Varadarajan, V S. “Has God Made the Quantum World P-Adic?,” n.d.
- Varadarajan, V. S. “Path Integrals for a Class of P-Adic Schrödinger Equations.” *Letters in Mathematical Physics* 39, no. 2 (January 1, 1997): 97–106. <https://doi.org/10.1023/A:1007364631796>.
- Weisbart, David. “ $p$ -Adic Brownian Motion Is a Scaling Limit.” *Journal of Physics A: Mathematical and Theoretical* 57, no. 20 (May 17, 2024): 205203. <https://doi.org/10.1088/1751-8121/ad40df>.