A dollar exchange model with bank and debt

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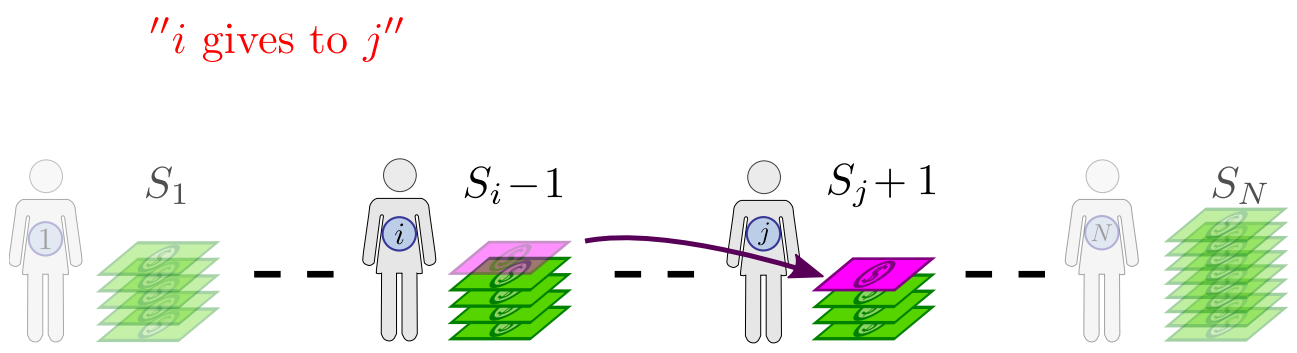
Settings: Unbiased dollar exchange model

Settings: N agents with money $S_i(t)$ for $t \geq 0$ and $1 \leq i \leq N$.

Dynamics: at random times (exponential clocks), pick i and j (uniformly). If i has a dollar (i.e., $S_i \geq 1$), i gives one dollar to j:

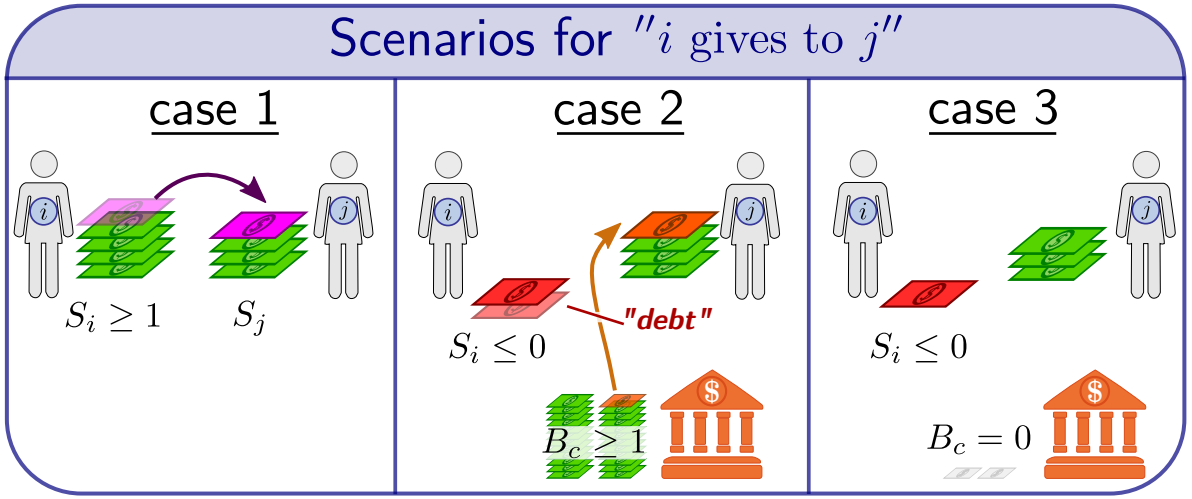
$$(S_i, S_j) \overset{\lambda}{\rightsquigarrow} (S_i - 1, S_j + 1) \quad (\text{if } S_i \geq 1).$$

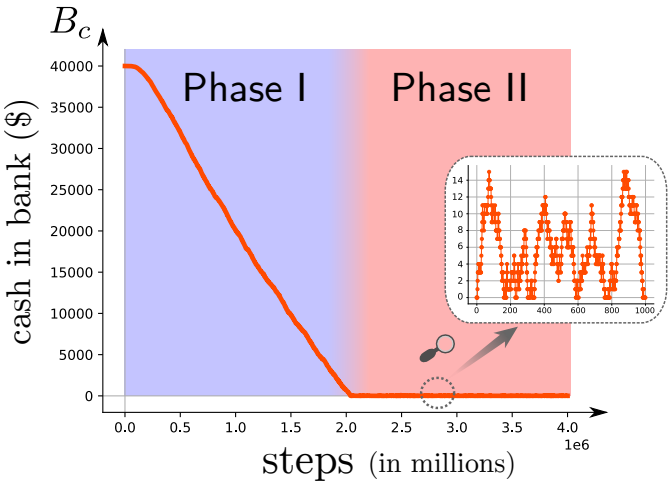
Illustration:

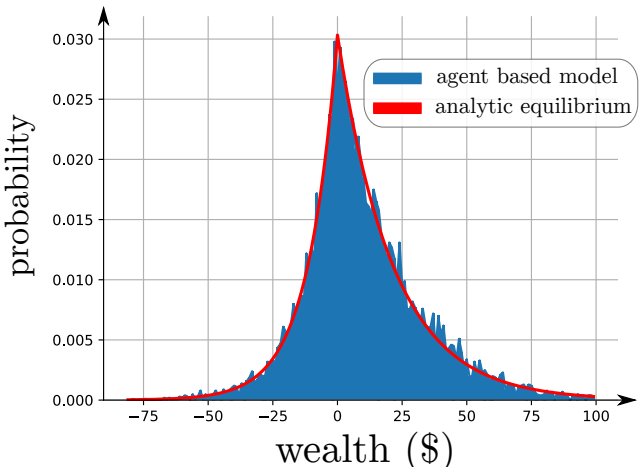


What happens as N and $t \to \infty$?

Settings: Unbiased exchange with collective debt limit







The two-phase dynamics as $N \to \infty$

As $N \to \infty$, a formal mean-field analysis yields the following ODE systems:

Phase I: $\partial_t \mathbf{p} = Q_1[\mathbf{p}]$ for $0 \le t \le t_*$,

Phase II: $\partial_t \mathbf{p} = Q_2[\mathbf{p}]$ for $t > t_*$,

and the evolution in Phase II will determine the equilibrium distribution of money.